

Learning Robot Manipulation Tasks with Task-Parameterized Semi-Tied Hidden Semi-Markov Model

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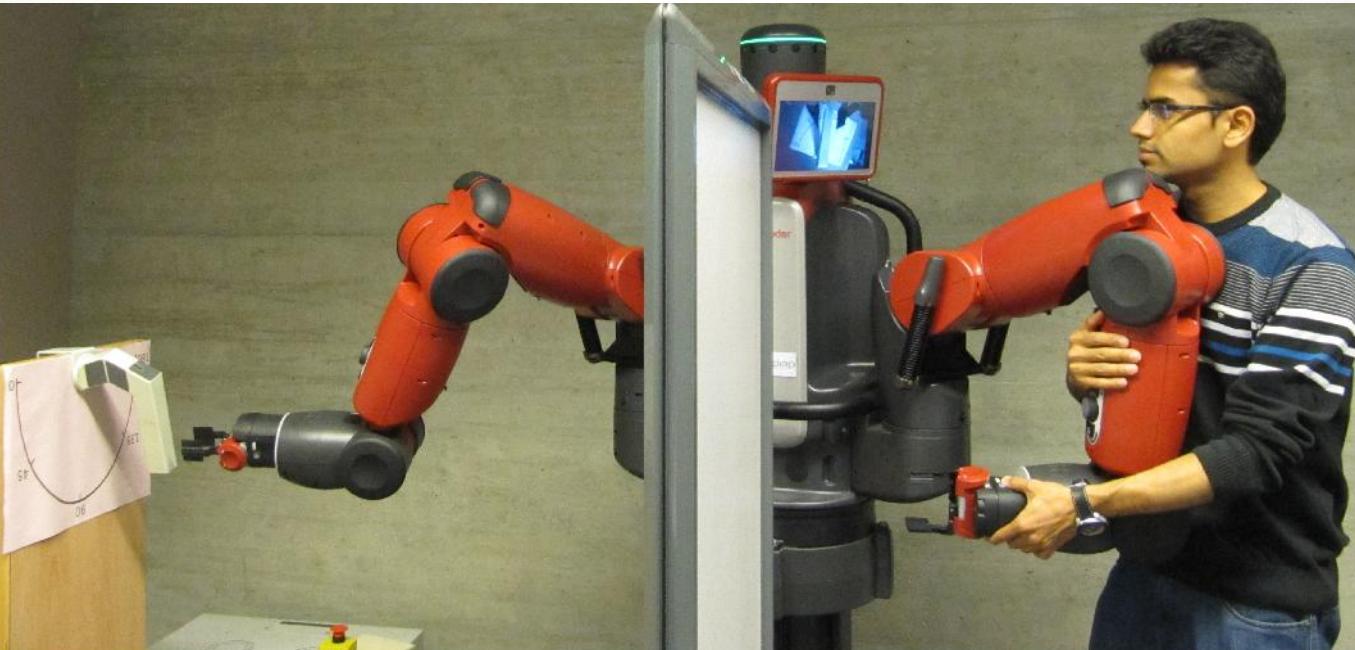
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Semi-Autonomous Teleoperation



Subspace
Clustering

Task
Adaptability

Trajectory
Reproduction

Recognition of intention
on teleoperator side

Reproduction of movement
on robot side

Problem Scope

$$\{\boldsymbol{\xi}_t \in \mathbb{R}^D\}_{t=1}^T \quad \mathcal{P}(\boldsymbol{\xi}_t | \theta) = \sum_{i=1}^K \pi_i \mathcal{N}(\boldsymbol{\xi}_t | \boldsymbol{\mu}_i, \Sigma_i)$$

Exploit **spatial** and **temporal correlation** in the demonstrations by **sharing parameters** across the mixture components

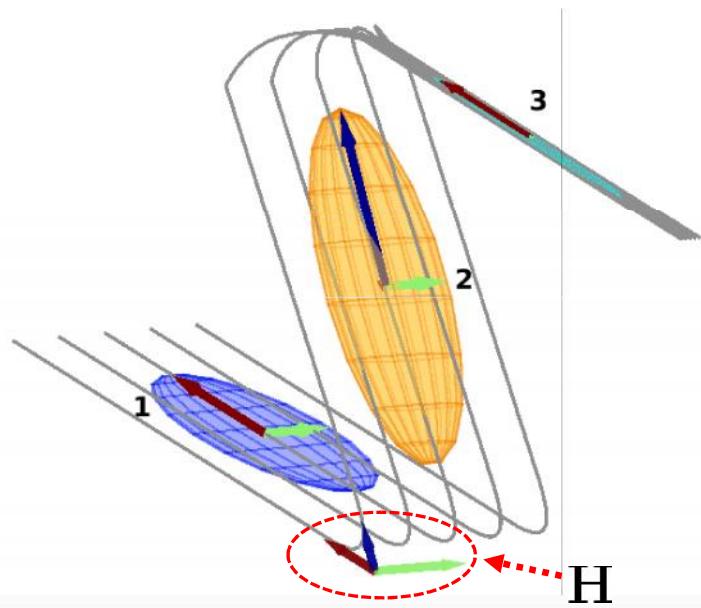
Adapt **movement** according to external environmental situation



Generative Model

$$\mathcal{P}(\boldsymbol{\xi}_t | \theta) = \sum_{i=1}^K \pi_i \mathcal{N}(\boldsymbol{\xi}_t | \boldsymbol{\mu}_i, \boldsymbol{\Sigma}_i)$$

Semi-tied GMMs $\boldsymbol{\Sigma}_i = \mathbf{H} \boldsymbol{\Sigma}_i^{(\text{diag})} \mathbf{H}^\top$



statistical decomposition of the covariance matrix to align the mixture components with similar coordination patterns for parsimonious representation and better generalization

$\mathbf{H} \in \mathbb{R}^{D \times D} \Rightarrow$ common latent basis vectors

$\boldsymbol{\Sigma}_i^{(\text{diag})} \in \mathbb{R}^{D \times D} \Rightarrow$ component-specific diagonal matrix

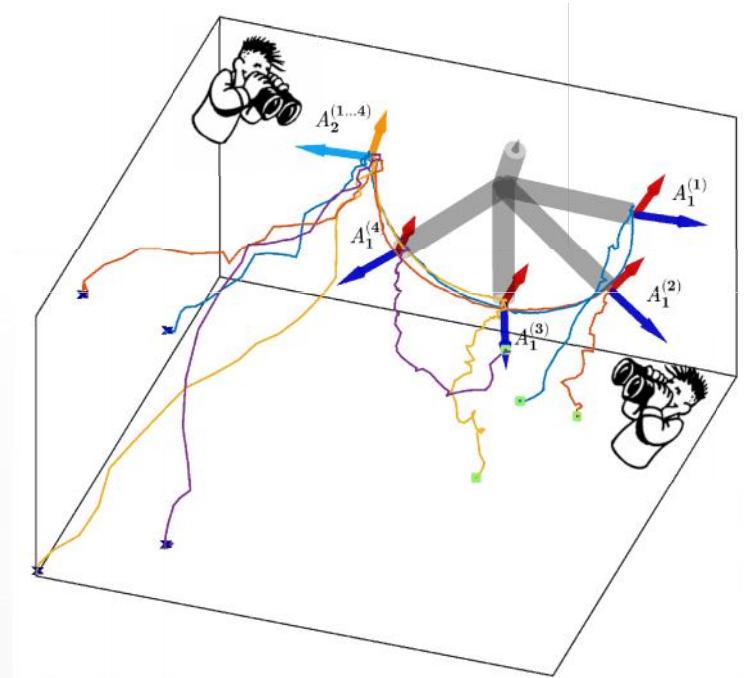
Generative Model

$$\{\mathbf{A}_j, \mathbf{b}_j\}_{j=1}^P : \{\boldsymbol{\xi}_t^{(j)}\}_{j=1}^P \quad \boldsymbol{\xi}_t^{(j)} = \mathbf{A}_j^{-1}(\boldsymbol{\xi}_t - \mathbf{b}_j)$$

Semi-tied GMMs $\Sigma_i = \mathbf{H}\Sigma_i^{(\text{diag})}\mathbf{H}^\top$

Task-parameterized semi-tied GMMs

observe the data from different frames of
reference/experts

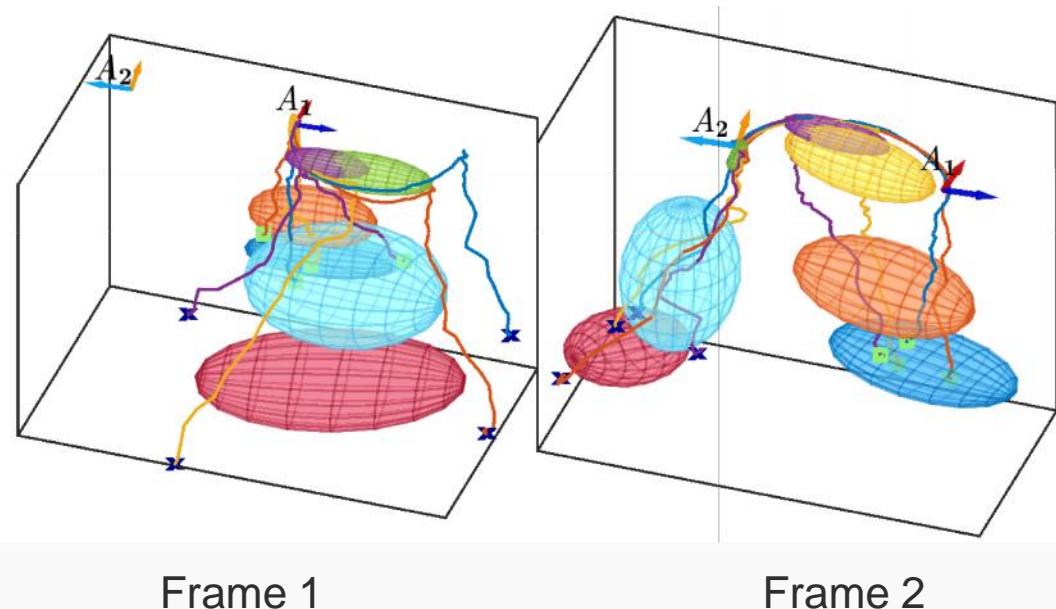


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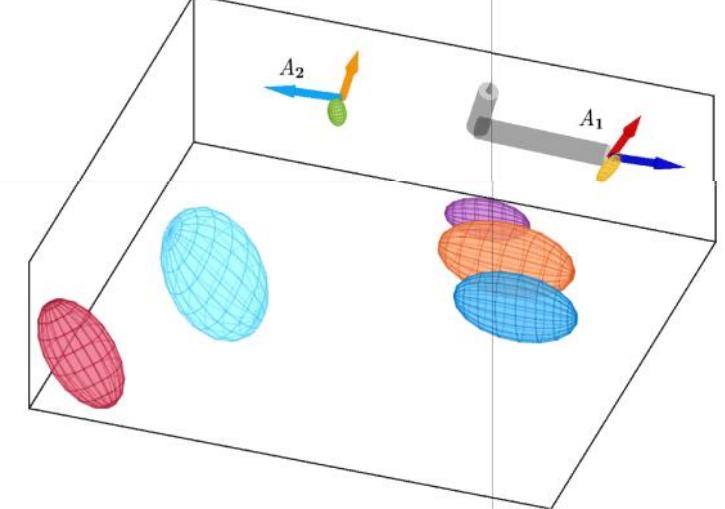
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Semi-tied GMMs $\Sigma_i = \mathbf{H}\Sigma_i^{(\text{diag})}\mathbf{H}^\top$

Task-parameterized semi-tied GMMs

take product of linearly transformed
Gaussians to adapt the model with
new environmental situation



$$\mathcal{N}(\tilde{\boldsymbol{\mu}}_i, \tilde{\Sigma}_i) \propto \prod_{j=1}^P \mathcal{N}\left(\tilde{\mathbf{A}}_j \boldsymbol{\mu}_i^{(j)} + \tilde{\mathbf{b}}_j, \tilde{\mathbf{A}}_j \Sigma_i^{(j)} \tilde{\mathbf{A}}_j^\top\right)$$

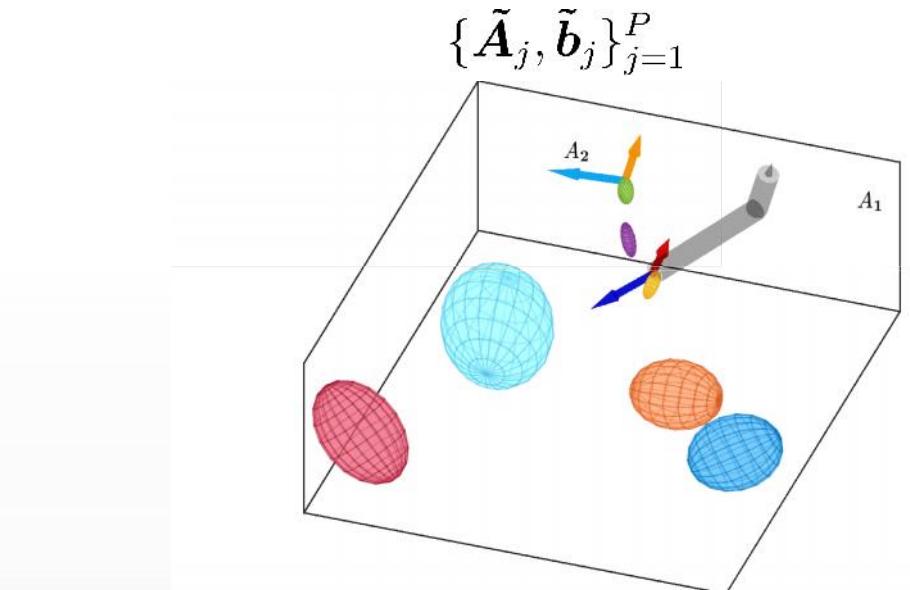
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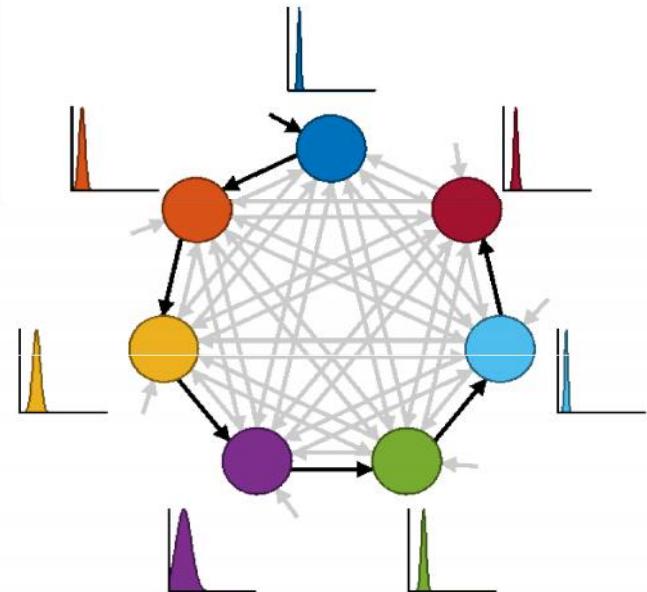
Generative Model

$$\theta_h = \left\{ \Pi_i, \{a_{i,m}\}_{m=1}^K, \{\boldsymbol{\mu}_i^{(j)}, \boldsymbol{\Sigma}_i^{(j)}\}_{j=1}^P, \mu_i^D, \boldsymbol{\Sigma}_i^D \right\}_{i=1}^K$$

Semi-tied GMMs $\boldsymbol{\Sigma}_i = \mathbf{H} \boldsymbol{\Sigma}_i^{(\text{diag})} \mathbf{H}^\top$

Task-parameterized semi-tied GMMs

Hidden semi-Markov model encoding

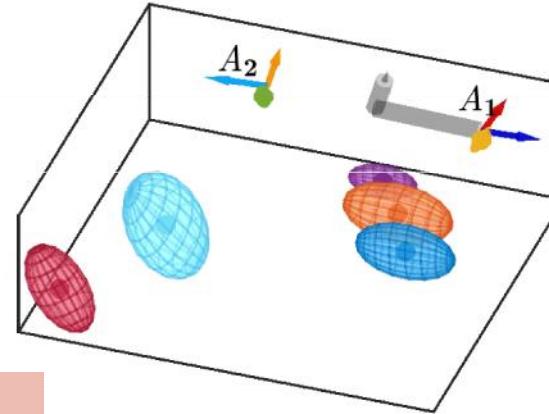


Generative Model

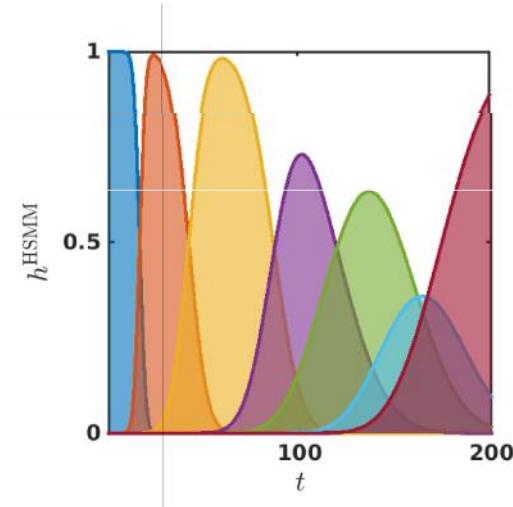
$$\alpha_{t,i}^{\text{HSMM}} = \sum_{j=1}^K \sum_{d=1}^{\min(d^{\max}, t-1)} \alpha_{t-d,j}^{\text{HSMM}} a_{j,i} \mathcal{N}(d | \mu_i^D, \Sigma_i^D)$$

Semi-tied GMMs $\Sigma_i = \mathbf{H} \Sigma_i^{(\text{diag})} \mathbf{H}^\top$

Task-parameterized semi-tied GMMs



Hidden semi-Markov model encoding



recognize the current state and plan the next step-wise sequence of states to be visited with the forward variable

$$q_t = \arg \max_i \alpha_{t,i}^{\text{HSMM}}, \quad \hat{\boldsymbol{\mu}}_t = \tilde{\boldsymbol{\mu}}_{q_t}, \quad \hat{\boldsymbol{\Sigma}}_t = \tilde{\boldsymbol{\Sigma}}_{q_t}$$

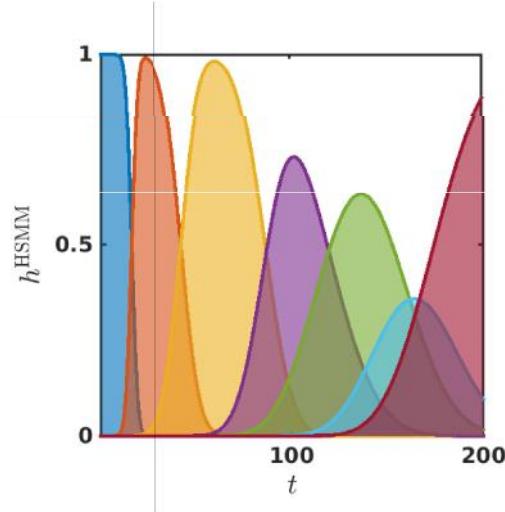
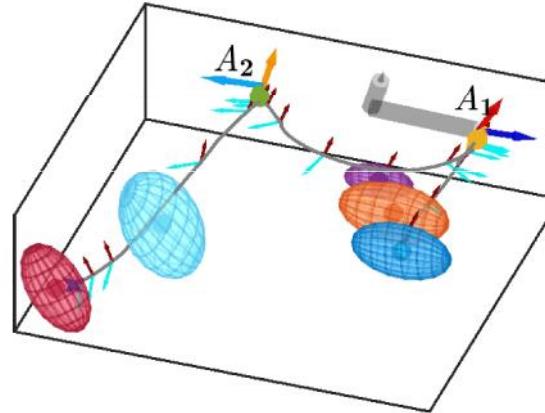
Generative Model

$$c_t(\xi_t, \mathbf{u}_t) = \sum_{t=1}^T (\xi_t - \hat{\mu}_t)^\top \hat{\Sigma}_t^{-1} (\xi_t - \hat{\mu}_t) + \mathbf{u}_t^\top \mathbf{R}_t \mathbf{u}_t$$

s.t. $\dot{\xi}_t = \mathbf{A}_d \xi_t + \mathbf{B}_d \mathbf{u}_t$

Semi-tied GMMs $\Sigma_i = \mathbf{H} \Sigma_i^{(\text{diag})} \mathbf{H}^\top$

Task-parameterized semi-tied GMMs



Hidden semi-Markov model encoding

Linear quadratic tracking controller

exploit the task variability to follow the desired sequence in a smooth manner

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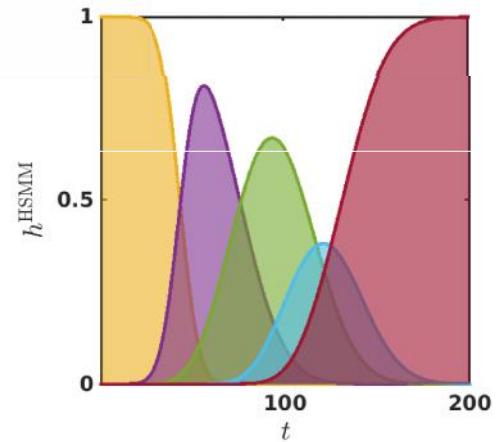
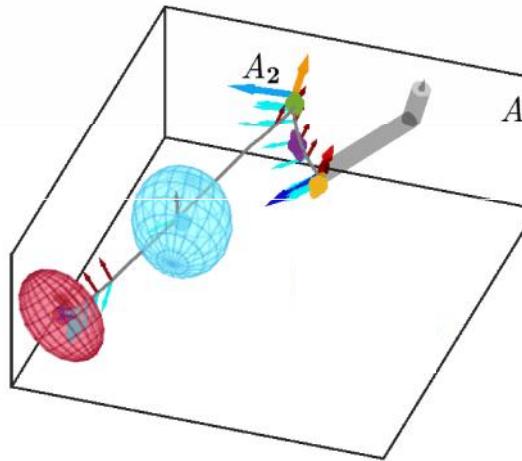
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Task-parameterized semi-tied GMMs

Hidden semi-Markov model encoding

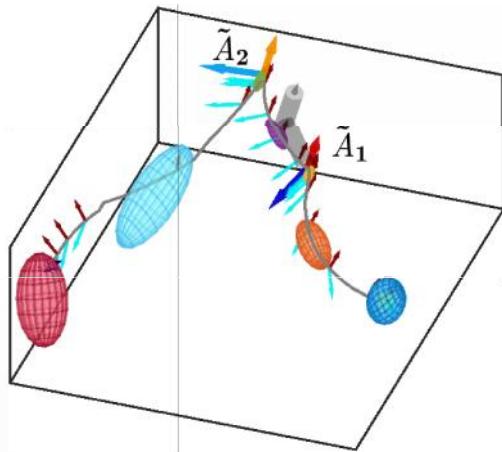
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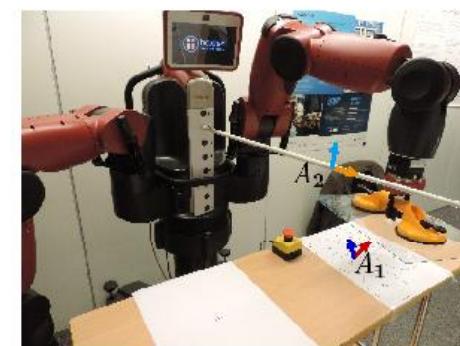
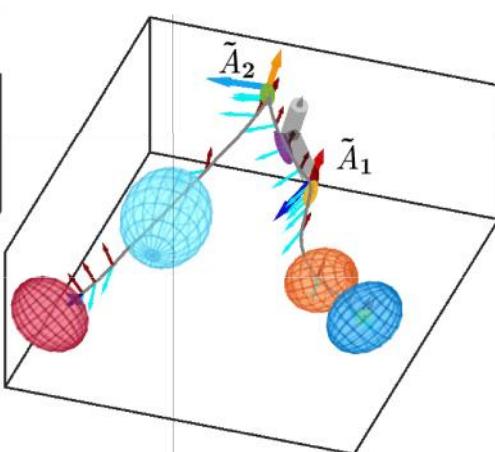


Experiments with Baxter

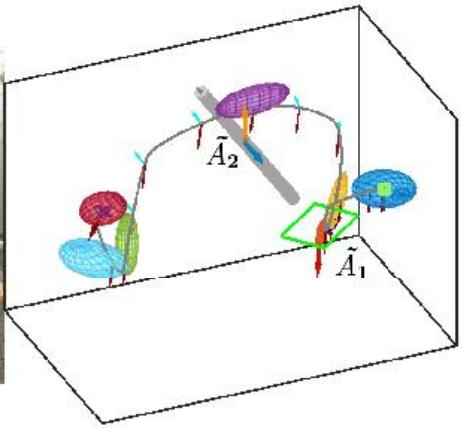
$$\xi_t = [x_t^{p\top} \ \boldsymbol{\varepsilon}_t^{o\top} \ \dot{x}_t^{p\top} \ \dot{\boldsymbol{\varepsilon}}_t^{o\top}]^\top \quad D = 14, K = 7, P = 2$$



Valve opening



Pick-and-place with obstacle avoidance



The model gives better generalization with much less parameters

The model readily adapts the movement to different manipulation scenarios

Conclusions

Semi-tied GMMs encode similar coordination patterns with a set of basis vectors/synergistic directions

Task-parameterized semi-tied HSMM autonomously adapts the model to changing environmental scenarios

